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GEOMETRY.

394. Proposed by W. J. GREENSTREET, M. A., Editor, The Mathematical Gazette, Stroud, England.

The joins of the excentres to the corresponding vertices of the pedal triangle are concurrent.

I. Solution by WILLIAM HOOVER, Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let $ABCD$ be the triangle of reference; p_1, the perpendicular AP_1 upon the side A, analogous notation making $P_1P_2P_3$ the pedal triangle. Using trilinear coördinates, the center C_1 of the escribed circle touching a is given by $(-\alpha_1, \alpha_1, \alpha_1)$, and similarly for the other centers.

The point P_1 is given by $(0, p_1\cos C, p_1\cos B)$, and P_2, P_3 are set down by symmetry.

The equation to P_1C_1 is

$$\begin{vmatrix} \alpha & \beta & \gamma \\ -\alpha_1 & \alpha_1 & \alpha_1 \\ 0 & p_1\cos C & p_1\cos B \end{vmatrix} = \begin{vmatrix} \alpha & \alpha+\beta & \alpha+\gamma \\ -1 & 0 & 0 \\ 0 & p_1\cos C & p_1\cos B \end{vmatrix}$$

$$= \begin{vmatrix} \alpha+\beta & \alpha+\gamma \\ \cos C & \cos B \end{vmatrix} = \alpha(\cos B-\cos C)+\beta\cos B-\gamma\cos C=0\ldots(1).$$

The equations to P_2C_2, P_3C_2 are, by symmetry,

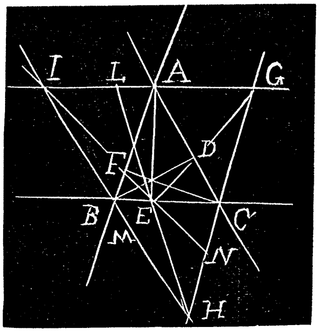
$$-\alpha\cos A+\beta(\cos C-\cos A)+\gamma\cos C=0\ldots(2),$$
$$\alpha\cos A-\beta\cos B+\gamma(\cos A-\cos B)=0\ldots(3).$$

(1), (2), and (3) meet in a point, since

$$\begin{vmatrix} \cos B-\cos C & \cos B & -\cos C \\ -\cos A & \cos C-\cos A & \cos C \\ \cos A & -\cos B & \cos A-\cos B \end{vmatrix}$$

$$= \begin{vmatrix} \cos B & \cos B & -\cos C \\ -\cos C-\cos A & \cos C-\cos A & \cos C \\ \cos B & -\cos B & \cos A-\cos B \end{vmatrix} =0.$$

II. Solution by N. L. RORAY.



Given the triangle ABC; D, E, and F vertices of the pedal triangle; G, H, and I centers of the ex-circles; GM, HL, and IN sects joining ex-centers with the corresponding vertices of the pedal triangle.

To prove: GM, HL, and IN concurrent.

Proof. $\dfrac{\triangle HIL}{\triangle BIA}=\dfrac{IL.IH}{IA.IB}$, $\dfrac{\triangle BIA}{\triangle HBE}=\dfrac{IB.BA}{BH.BE}$.

$$\therefore \frac{\triangle HIL}{\triangle HBE}=\frac{IL.1H.BA}{IA.BH.BE}=\frac{HI.HL}{HB.HE}. \qquad \therefore \frac{IL}{IA}\cdot\frac{BA}{BE}=\frac{HL}{HE}.$$

Similarly, $\frac{GA}{GL}\cdot\frac{CE}{AC}=\frac{HE}{HL}$; $\frac{GN}{GC}\cdot\frac{AC}{AF}=\frac{IN}{IF}$; $\frac{HC}{HN}\cdot\frac{BF}{BC}=\frac{IF}{IN}$;

$\frac{IB}{IM}\cdot\frac{AD}{AB}=\frac{GD}{GM}$; and $\frac{HM}{HB}\cdot\frac{BC}{CD}=\frac{GM}{GD}$.

$$\therefore \frac{IL.GN.HM}{GL.HN.IM}\cdot\frac{GA.IB.HC}{IA.BH.GC}\cdot\frac{CE.BF.AD}{BE.CD.AF}=1.$$

But by Ceva's Theorem, $\frac{CE.BF.AD}{BE.CD.AF}=1$.

Also, it is easily proved that $\frac{GA.IB.HC}{IA.BH.GC}=1$. $\therefore \frac{IL.GN.HM}{GL.HN.IM}=1$.

$\therefore$ *GM*, *HL*, and *IN* are concurrent. Q. E. D.

397. Proposed by DAVID F. KELLEY, New York City.

If *ABC* be a semicircle and *CD* a perpendicular from *C* on the diameter *AB*, prove that the radius of the circle inscribed in the triangle *ABC* equals half the sum of the radius of the circle touching arc *AC* and the sides *AD* and *DC* of the triangle *ADC*, and the radius of the circle touching arc *CB* and sides *DB* and *DC* of triangle *CDB*, and that the centers of the three circles are collinear.

I. Solution by H. C. FEEMSTER, Professor of Mathematics, York College, York, Nebraska.

Let *AB* be the y-axis, *CD* the x-axis, r the radius of the given circle, $k=r-AD$.

Then $\begin{cases} x_1^2+(x_1+k)^2=(r-x_1)^2 \\ y_1=x_1, \end{cases}$ (1),

and $\begin{cases} x_{11}^2+(k-x_{11})^2=(r-x_{11})^2 \\ y_{11}=-x_{11}, \end{cases}$ (2),

are the relations that give the centers of the circles inscribed in *ADC* and *BDC*. The bisectors of the angles *A* and *C* intersect at

$$x=\frac{-2r+\sqrt{(2r^2-2rk)}+\sqrt{(2r^2+2rk)}}{2},$$

$$y=\frac{-2k+\sqrt{(2r^2+2rk)}-\sqrt{(2r^2-2rk)}}{2},$$

the center of the circle inscribed in *ABC*.